L'elicoide e le sue applicazioni in architettura
Helicoid and Architectural application

Laura Inzerillo, Engineering School, Department of Architecture, University of Palermo.

Abstract

Questo articolo è un risultato parziale di una ricerca riguardante la rappresentazione di superfici complesse in geometria descrittiva. La padronanza e l'abilità nell'uso delle diverse tecniche di rappresentazione, consente di raggiungere risultati che altrimenti non sarebbero perseguibili. La Scuola di Disegno di Ingegneria, dell'Università di Palermo si è da sempre fatta promotrice della scienza della rappresentazione attraverso la sperimentazione di tecniche semplificative ed innovative della geometria descrittiva applicata all'architettura e all'ingegneria. In particolar modo, in questo lavoro, si riporta lo studio di una superficie complessa quale è l'elicoide che trova larga applicazione nell'architettura. L'elicoide è qui trattata nella rappresentazione dell'assonometria ortogonale. Il metodo proposto è basato fondamentalmente sull'applicazione indispensabile ed imprescindibile dell'omologia. Alcuni passi, qui dati per scontato, trovano riscontro nei riferimenti bibliografici.

This paper presents the issue of a long research on the representation of the complex surface in descriptive geometry. The ability to use the different techniques of representation aims to achieve results that you didn't image before. In Palermo University, at the Engineering School, the researcher involved the study on the simplify of the so elaborated way to represent the geometry and its applications in architecture buildings and engineering implants. There is just report below the application methods to represent one of the most used surfaces in the practice of buildings. It's about the helicoid surface represented in orthogonal axonometric projecting. This method is the result of the experimental homology. I remained to the bibliography of the other works where there is explained the theory method to become to the application.

Keywords: Descriptive geometry, graphics, homology, helicoid, orthogonal axonometric

Introduction

In the below application of the axonometric projection it is applied the homology method to obtain from the turnover of the real geometry figure in scale to the projection of it. The method, issue of the research, is the simplification of the traditional axonometric method projection and is focused on the turnover homology in which we have the axis of homology in the trace of the plane of the figure and the centre of the homology at infinite point in orthogonal direction respect the trace. This is possible to consider because the projection is orthogonal and the turnover is applied around the trace of the plane. So we will have the projection point and the turnover point aligned into the orthogonal direction from the trace line. The points, the one limelight and the one projected are placed in two correspondent lines that intersect themselves on the trace line. This is the major simplification that we can do to have a correct representation with the minor effort.

Helicoid

The helicoid, after the plane and the catenoid, is the third minimal surface to be known. It was first discovered by Jean Baptiste Meusnier in 1776. Its name derives from its similarity to the helix: for every point on the helicoid there is a helix contained in the helicoid which passes through that point. Since it is considered that the planar range extends through negative and positive infinity, close observation shows the appearance of two parallel or mirror planes in the sense that if the slope of one plane is traced, the co-plane can be seen to be bypassed or skipped, though in actuality the co-plane is also traced from the opposite perspective. The helicoid is also a ruled surface (1, and a right conoid), meaning that it is a trace of a line. Alternatively, for any point on the surface, there is a line on the surface passing through it. Indeed, Catalan proved in 1842 that the helicoid and the plane were the only ruled minimal surfaces. The helicoid and the catenoid are parts of a family of helicoid-catenoid minimal surfaces. The helicoid is shaped like Archimedes' screw, but extends infinitely in all directions. It can be described by the following parametric equations in Cartesian coordinates:

$$x = \rho \cos (\alpha\vartheta) \qquad (0)$$

$$y = \rho \sin (\alpha\vartheta) \qquad (1)$$

$$z = \vartheta \qquad (2)$$

where ρ and ϑ range from negative infinity to positive infinity, while α is a constant. If α is positive then the helicoid is right-handed as shown in the figure; if negative then left-handed.

The helicoid in descriptive geometry

We consider a directrix of the motion that can be a straight axis or an helic - sometimes cylinder - or a no plane curve - that belongs to a surface - also quadric surface, also open to infinity also wreathed. The generatrix can be made of one or more than one segment, of a plan or not plane polygonal, an open or closed curves, a plane curve or a bent curve, a simply or not simply curve, a single or multiple curve, a modular curve, etc. The rototranslation motion can affect the final results in function to the uniform or not uniform ruled speed, in function to the translation speed, in function to the relationship between the ruled and translation speed. In figure 2 there is an helicoid that's the shape for a stair, delimited to a perimeter girder, which height is triple of the rise height; the vertical shapes are two cylinder for each girder; the shape of the extrados is a helicoid surface. Between the vertical shaped there are the steps which vertical plane risers belong to the axial plane and which the horizontal tread are not rectangular but are defined by two concentric arches and two complanar straight lines that converge to the axis. The tread depth can be measured on the average circumference, projection on the horizontal plane of the average helic; the ratio rise height/ tread depth is about in 16/30. In some way this sets the stair slope and is in the range of the maximum value in the interior edge of the rise to a minimum in the exterior edge. Where's possible we can assume a great radius for the stairwell and if it is not possible we can established that the rise plane not be axial but tangent to a virtual coaxial cylinder. The runnel stair is the surface of the helicoid flight that we can admire from the horizontal plane of the beginning corresponding to the second rise and is delimited from to coaxial and cylinder helics that are on the circumferences of radius 0_{XY}-1 and 0_{XY}-2, major semi-axis of the corresponding ellipses the maximum- external one and the minimum- internal one; we have the corresponding minor semi-axes homology with centre at infinite in H-O* direction.

Draw, now, the four ellipses that are the projection of other four circumferences, traces on xy, of the director cylinder of the helics, two for the runnel stair and two of the upper edge of the girders; the projection on π of the helic is a cycloid; the vertical pitch 0_{XY}-3 divided in 20 equal parts is the projection of the helic pitch, shortened according z*. The drawing of a second flight of stairs is obtained by translation of a step along the axis, equal to 0_{XY}-3; the fulfillment of the step, we climb 20 steps; so divide into 20 equal parts the outer circumference, homological to the outer ellipse.

To avoid in projection the result not very effective of two treads on the projecting axial plane, the subdivision is rotated, so the ends of the axes aren't subdivision points. From the homological circumference, with the vertical of reference, we obtain the subdivision of the major ellipse and by homothety with center 0_{XY} that of the other ellipses; the twenty arcs don't look alike, but correspond to arcs equal to each other. The initial radius is 0_{XY}-4-5; the edge of the first tread 6-7 converges into 8 and comes from the points 9 and 10, belonging to the ellipse relative to the width of the steps; the vertexes 11-12 and 13-14 of the edges of the beams converges onto 15 and come from the points 5-9 and 10-4. On xy between 5-0_{XY} and 16-0_{XY} we have two arcs in 9-17 and 10-18; we lead them onto a riser equal to 0_{XY}-8; the first isn't in view and is omitted, the second translates into 7-19; the bottom edge of the first tread is exposed only for the stretch 19-20. It's fundamental the correct observation of what is in view and what is not; the edges of the risers are parallel, the ones of the tread are convergent; risers are all the same height equal to 0_{XY}-8, while the treads are all different.

Since the subdivision is rotated, the axis doesn't work as an axis of relative symmetry; a riser with full width does not exist; each riser takes two points of the axis, at the next height, but one unit above and one unit below we retrieve points of the edges of beams and runnel.

The outer cylindrical wall shows the base arc, that is circular, but elliptical in projection, 5-16 and from 16 to 22, with projection 21, that is the end of the major semiaxis, is the projection of the helix arcs; 20-23 is projection of arc of helix; 23-24 is the translation of 16-22; the vertical segment 22-24 is projection of the generatrix of the apparent contour of the cylindrical area between two helices translated. This area beyond 22-24 is no more in view; it reappears from the vertical stretch 25-26, on the vertical of the end 1 to end in 27-28; 16-22 continues to be not in view and reappears from 25 to 27, while 20-23-24 continues in view up to 29, reappearing in 30-26-28. The projection of the helix in the space, starting from 12, maintains a constant thickness, equal to 1-31, with previous situation which starts from the point 11; in projection this difference between the pertinent radii varies with the rotation, disappears from the view in 32, reappears in 33, in 34 is on the vertical through 31, and is in view from 34 to 35. From the observation of the projections of the two superior helices side by side, belonging to the double inner director cylinder, we note the generatrix of apparent contour 36-37, which is two units high, whose foot 36 doesn't coincide with the vertex 38 of the edge of the fourth step and on the tread there is, although difficult to be seen, the arc 38-36, which is part of the corresponding arc on xy plane. We also note that, the two projections, the inner one, to which 36-37 belongs, from 39 are no more in view and reappeas in 40, ending in 41; in the area not in view, the aforesaid curve is tangent to the vertical through 42; then in 43 it shows a flex and the thickness is projecting over π. The curve that starts from 14, is in view up to 39, continues in the not in view area, where it is tangent to the vertical through 2, and reappears in 43, ending in 44. The curve that starts from 4 has the first stretch not in view composed of an arc belonging to the base ellipse and continues with the projection of the first stretch of the helix; it is in view in the stretch 45-46, showing in 47 an inflection point, on the vertical through 43; the other is in the hidden stretch, ending in 48; this curve is inferior edge of the cylindrical face with minor radius and it internally delimits the helicoidal runnel. The curve that starts from 16 is in view up to 22, continues hidden up to 47, reappears for the stretch 47-49, reappears, creating a loop, for the stretch 25- 27; the helicoidal runnel surface is visible only by the triangle 47-46-49. Prompting at the highest level, the observation of results is possible by highlight, so representing a way of an easy control, all resulting from the base radius and the height of verification; changing the homological ratio, or realizing only an helicoid without the steps, we can certainly better understand the predictability of the genesis and the final view. So, in fig. 2, is a 3D model of a spiral staircase. In the same way, just like a real spiral stairs, this 3D staircase revolves around a central pole. A pure spiral generally assumes a circular stairwell. The steps and handrail are equal and positioned screw-symmetrically. So, those are the features of any spiral staircase even if it is 3D modelled. Therefore, talking of this particular spiral staircase 3D model, one should argue that it is quite tight what makes this staircase 3D model very space efficient in the use of the floor area.

The helicoid in Architecture.

In Architecture, the most popular use of the helicoid is in the staircase. Spiral stairs turn around a newel (also the central pole). They typically have a handrail on the outer side,, showing on the inner side just the central pole. A pure spiral assumes a circular stairwell and the steps and handrail are equal and positioned screw-symmetrically.

A tight spiral stair with a central pole is very space efficient in the use of floor area. Spiral stairs have the disadvantage of being very steeped. Even in the case of a very large central column, the circumference of the circle at the walk line will be small enough to avoid a normal tread depth and a normal rise height without compromising headroom before reaching the upper floor. To guarantee headroom, most spiral stairs have very high rises and a very short going. Most building codes limit the use of spiral stairs to small areas or to a secondary usage.

The term "spiral" is used incorrectly for a staircase from a mathematical point of view, as a mathematical spiral lies in a single plane and moves towards or away from a central point. A spiral staircase by the mathematical definition therefore would be of little use as it would afford no change in elevation. The correct mathematical term for motion where the locus remains at a fixed distance from a fixed line, whilst moving in a circular motion about it, is actually "helical". The presence or not of a central pole does not affect the terminology applied to the design of the structure.

Ancient buildings

Spiral stairs in medieval times were generally made of stone and typically wound in a clockwise direction (from the ascender's point of view), to place attacking swordsmen (who were most often right-handed) at a disadvantage. This asymmetry forces the right-handed swordsman to engage the central pike and degrade his mobility compared with the defender who is facing down the stairs.

Exceptions to this rule however exist, as it may be seen in the accompanying image of the Scala of the Palazzo Contarini del Bovolo (fig. 3.b), which winds up counter-clockwise. Developments in manufacturing and design have led to the introduction of kit form spiral stairs. Steps and handrails can be bolted together to form a complete unit. These stairs can be made out of steel, timber, concrete or a combination of materials. Helical or circular stairs do not have a central pole and there is a handrail on both sides. These have the advantage of a more uniform tread width when compared to the spiral staircase. Such stairs may also be built around an elliptical or oval plane form. A double helix is possible, with two independent helical stairs in the same vertical space, allowing one person to ascend and another to descend, without ever meeting if they choose different helices like in Château de Chambord (fig. 3.a-4.a), Château de Blois (fig. 3.c), Crédit Lyonnais headquarters in Paris (fig. 4.b). Fire escapes, though built with landings and straight runs of stairs, are often functionally double helices, with two separate stairs intertwined and occupying the same floor space. This is often in support of legal requirements, in order to have two separate fire escapes. Both spiral and helical stairs can be characterized by the number of turns that they are made by. A "quarter-turn" stair deposits the person facing 90 degrees from the starting orientation. Likewise there are half-turn, three-quarters-turn and full-turn stairs. A continuous spiral may make many turns depending on the height. Very tall multi-turn spiral staircases are usually found in old stone towers within fortifications, churches and in lighthouses. Winders may be used in combination with straight stairs to turn the direction of the stairs. This allows for a large number of permutations. Another amazing sight offered at the Abbey of Melk, lower Austria, is the spiral staircase that connects its library to church (fig. 4).

But also The Vatican Museums spiral staircase is one of the most photographed in the world, and certainly one of the most beautiful (fig. 5). Designed by Giuseppe Momo in 1932, the broad steps are somewhere between a ramp and a staircase. The stairs are actually two separate helixes, one leading up and the other leading down, that twist together in a double helix formation.

Padula Charterhouse, in Italian Certosa di Padula (or Certosa di San Lorenzo di Padula), is a large Carthusian monastery, or charterhouse, located in the town of Padula, in the Cilento National Park (near Salerno), in Southern Italy. Inside the building there is a spiral staircase, belonging to a secondary part of the structure (fig. 4.a).

In Torgau Hartenfels Castle there is the most significant building of the townscape, an outstanding masterpiece in one of Germany's most beautiful Renaissance towns. It is dominated by the Great Wendelstein Staircase (fig. 5.b): this "impossible staircase" of the great architect Konrad Krebs supports itself without the aid of any central pillar. One of the most interesting triple spiral staircase is the one inside the Museo do Pobo Galego, Santiago de Compostela, Spain (figgs. 5.c, 5.d). These staircases create an unusual triple helix, that is an intentionally mystifying feature, allegedly designed to confuse unauthorized visitors to this former convent.

Conclusions

The aim of this work is to illustrate the importance to know the descriptive geometry for the conservation and restoration of ancient buildings. At this end, I have chosen "the helicoidal surface", which application in the architectural building is very impressive. The well-known knowledge of the descriptive geometry is fundamental to well understand and well represent the cultural heritage.

In the present work, I considered some architectural buildings applications and how the geometry complex surface is easy to analyze if it is well-known under the analytic aspect. The application illustrates the applicability of helicoidal surface both elliptical and circular. In detail, if the researcher doesn't know the potentially of descriptive geometry he can't well understand the real geometry of the surface and the representation by Computer Aided Design is not well done. Based on this considerations, the research concerning these new representative methods based on homology and prove how easy is the axonometric representation by homology use.

Notes

1 - In geometry, a surface S is ruled (also called a scroll) if through every point of S there is a straight line that lies on S. The most familiar examples are the plane and the curved surface of a cylinder or cone. Other examples are a conical surface with elliptical directrix, the right conoid, the helicoid, and the tangent develop able of a smooth curve in space. A ruled surface can always be described (at least locally) as the set of points swept by a moving straight line. For example, a cone is formed by keeping one point of a line fixed whilst moving another point along a circle. A surface is doubly ruled if through every one of its points there are two distinct lines that lie on the surface. The hyperbolic paraboloid and the hyperboloid of one sheet are doubly ruled surfaces. The plane is the only surface which contains three distinct lines through each of its points.

References

Barrett, J., "Push to Finish Tallest Tower". The Wall Street Journal. Retrieved 2009-12-10.

Mapes, T., "The Turning Torso in Malmo, Sweden". About.com. Retrieved 2009-02-02.

Fomenko, A. T., (1991). Elements of the Geometry and Topology of Minimal Surfaces in Three-dimensional Space, in A. A. Tuzhilin Contributor A. A. Tuzhilin Published by AMS Bookstore, p.33

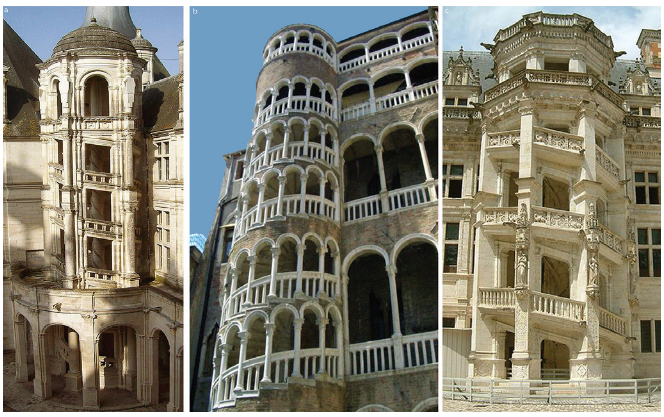
Neufert, E., Neufert, P., (2000). Architects' Data (3rd ed.). Blackwell Science, 2000, p. 191

Inzerillo, L., (2002). Assonometria Diretta, in L. Carnevalis-L. De Carlo- R. Migliari (a cura di) "Attualità della geometria descrittiva", Cangemi Editore, Roma.

Di Paola, F., (2010). Le Curve di Apollonio. Tradizione ed Innovazione nei processi risolutivi, Aracne Editrice, Roma.

Inzerillo, L., (2012). Assonometria e futuro. Vol. Unico, p. 1-200, Palermo: Aracne Editrice.

Figures 1. Guggenheim Museum, NY.



Figures 3. 3.a: Château de Chambord, France; 3.b: Palazzo Contarini del Bovolo, Italy; 3.c: Château de Blois, France.

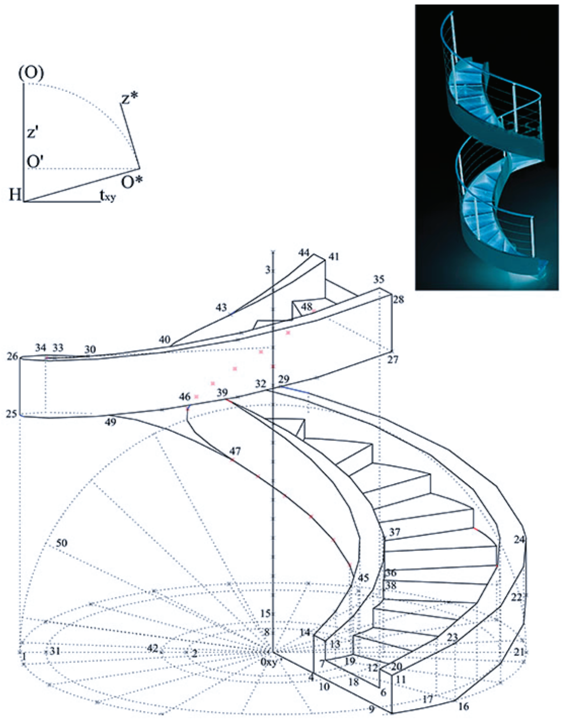


Figure 2. Orthogonal axonometric projection of a helicoidal stair.

Figures 4. 4.a: Château de Chambord, France; 4.b: Crédit Lyonnais headquarters, Paris, France; below Staircase in Melk Abbey, Austria.

Figures 5. Staircase in Vatican Museum, Italy; below: a) Staircase in Padula Charterhouse, Italy; b) spiral staircase in Hartenfels Castle, Torgau, Germany; c)- d) Museo do Pobo Galego, Santiago de Compostela, Spain.